HYDRAULIC PERFORMANCE OF LABYRINTH WEIRS FOR HIGH HEADWATER RATIOS

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Abstract: Physical and numerical modelling were used to investigate the hydraulic performance of labyrinth weirs operating under high headwater ratios (greater than 1). Physical modelling was conducted in a rectangular flume in a laboratory setting. Numerical modelling was conducted with commercially available computational flow dynamic (CFD) software (Flow-3D). Preliminary results indicate that the CFD model can accurately predict the labyrinth weir head-discharge relationship obtained from the physical model, including upstream heads (relative to the crest) that exceed the weir height. Furthermore, the information presented in this paper indicates that the hydraulic design curves developed by Crookston in 2010 may be acceptable for headwater ratios up to 2.0 or more.

Keywords: Labyrinth weirs, High headwater ratio, Physical models, Numerical models, Spillways

INTRODUCTION

A labyrinth weir is a passive control structure that is 'folded' in plan-view (Fig. 1) to increase the crest length, thereby increasing the discharge capacity relative to linear overflow control structures for a given channel width and upstream head. Due to their hydraulic efficiency, labyrinth weirs are often a favourable design option to increase flow capacity (e.g., spillway rehabilitation, replacement, new spillways) and regulate upstream water elevations (e.g., limited freeboard, flooding, water level control). The authors have identified more than 60 labyrinth weir prototype structures throughout the world.

Background

Labyrinth weirs produce complex, three-dimensional flows that are not readily described analytically. As is commonly done with hydraulic structures, empirical relationships that utilize a discharge coefficient (ascertained experimentally) have been developed to determine the head-discharge relationship of labyrinth weirs. This study utilizes equation (1) to determine the head-discharge relationship (Crookston, 2010); Q is the labyrinth weir flow rate, $C_{d(\alpha°)}$ is a dimensionless discharge coefficient, L_c is the centerline length of the weir crest, g is the acceleration constant of

Fig. 1 – Photograph of the construction of the 7-cycle labyrinth spillway at Lake Townsend (North Carolina, USA)

gravity, and H_T is the total upstream head defined as $H_T = V^2/2g + h$. V is the average cross-sectional velocity and h is the piezometric head upstream of the weir relative to the weir crest elevation. $C_{d(\alpha°)}$ is a function of geometry [sidewall angle (α), crest shape (e.g., quarter-round, half round, flat, truncated ogee, WES), weir height (P), cycle width (w), cycle depth (B), apex geometry, cycle orientation (i.e., linear, arced)], approach flow conditions, nappe behaviour (aeration condition, nappe instability), local submergence, and tailwater submergence.

$$Q = \frac{2}{3} C_{d(\alpha°)} L_c \sqrt{2g} H_T^{3/2} \tag{1}$$

Over the past 70 years, numerous research studies, case studies, and design methods have been published that have advanced hydraulic understanding of labyrinth weirs. Taylor (1968), Hay and Taylor (1970), Darvas (1971), Hinchliff and Houston (1984), Lux and Hinchliff (1985), Magalhães and Lorena (1989), Tullis et al. (1995), Melo et al. (2002), Tullis et al. (2007) are a selection of notable investigations conducted with physical models to quantify the hydraulic behaviours of labyrinth weirs, with emphasis on geometric and hydraulic influences on discharge capacity. Several dimensionless ratios are used to describe and quantify complex labyrinth flows and the influences of geometric parameters [e.g., headwater ratio (H_T/P), cycle width ratio (w/P), magnification ratio (L/W), cycle efficiency (ε')]. Lopes et al. (2006, 2008), Wormleaton et al. (1998) and Emiroglu et al. (2010) are examples of labyrinth weir research focused upon energy dissipation, downstream flow characteristics, and aeration. Recent research efforts have included numerical modelling to evaluate and validate the use of CFD algorithms as an additional design tool (Savage et al. 2004, Paxson and Savage, 2006).

Headwater Ratio

As noted herein, labyrinth spillways can pass significantly larger discharges than a linear weir having equivalent channel widths (W); however, unlike typical linear weirs, $C_{d(\alpha°)}$ decreases with increasing headwater ratios (H_T/P). It is likely that the discharge capacity of a labyrinth weir will approach that of a broad crested weir (crest length equal to W). However, this theory has not been experimentally proven and the value of H_T/P where this occurs been identified.

Previous studies (Tullis et al. 1995, Lux and Hinchliff, 1985, Magalhães and Lorena, 1989, Melo et al. 2002, Crookston 2010) include experimental data sets that reach maximum H_T/P values of approximately 0.7 to 0.9. A review of the $C_{d(\alpha°)}$ values developed by Crookston (2010) for quarter round trapezoidal labyrinth weirs indicates that for a weir with α = 12° and H_T/P = 0.85, $C_{d(12°)}$ is equal to about 0.32. For a linear weir (α = 90°) with a quarter round crest and H_T/P = 0.85, the same figure can be used to obtain a $C_{d(90°)}$ value of about 0.74. The 12 degree labyrinth has a weir length equal to about four times the total spillway width; therefore, the discharge coefficient of the labyrinth could be expressed in terms of the spillway width and would be equal to 1.28, or 73% greater than the linear weir. This implies that even for the maximum tested H_T/P labyrinth weirs are still capable of passing more flow per unit width than linear weirs.

Tullis et al. (1995) developed polynomial equations to compute $C_{d(\alpha°)}$ values for various labyrinth geometries and H_T/P ratios. Application of these equations for H_T/P greater than about 0.9 produces erroneous results. However, the curve-fit equations developed by Crookston (2010) appear to accurately follow $C_{d(\alpha°)}$ trends and appear to predict the hydraulic performance of labyrinth weirs at H_T/P values of 1.0 and larger.

Determining the hydraulic performance of labyrinth weirs for H_T/P values greater than 1.0 was considered beneficial for the following reasons:

1) Previous published data was limited to maximum H_T/P values of approximately 0.9, despite the significantly higher discharge capacity of the labyrinth weirs when compared to linear weirs.
2) In practical applications, site conditions may limit the ability to construct a labyrinth spillway with a weir tall enough to keep within the range of H_T/P values for the design discharge.
3) Although the equations for computing $C_{d(\alpha°)}$ by Crookston (2010) appear suitable for these higher H_T/P values, the accuracy of these equations in this range needs to be determined.

The purpose of this study was to evaluate the hydraulic performance of labyrinth weirs for H_T/P values greater than the maximum values tested in previous publications, thus expanding the available H_T/P design range. In addition to physical modelling, this study incorporated numerical modelling to provide further validation of the application of CFD algorithms to evaluate the discharge characteristics of labyrinth weirs. The preliminary results of this study are presented herein.

EXPERIMENTAL METHOD

Physical Modeling

Physical modelling was conducted at the Utah Water Research Laboratory (UWRL) located in Logan, Utah, USA. The tested trapezoidal labyrinth weirs are summarized in Table 1. The models were fabricated from high density polyethylene (HDPE) plastic and tested in a rectangular flume (1.2 m wide x 14.6 m long x 1 m deep).

Table 1 – Physical models summary

	α (°)	P (mm)	$L_{c\text{-}cycle}$ (m)	w/P	N	Crest Shape	H_T/P
1	15	152.4	0.996	2.0	4	Quarter Round	0.32 – 2.10
2	15	304.8	1.991	2.0	2	Quarter Round	0.05 – 0.93

Model flow rates were metered using calibrated orifice meters in the flume supply piping, differential pressure transducers, and a data logger. Uniform approach conditions were created via a headbox and baffle assembly. A stilling well and point gage (±0.15 mm) assembly were used to measure head upstream of the labyrinth weir. Experimental data were collected under steady-state conditions. Hydraulic investigations also included nappe profiling and dye injection. Testing was documented extensively with digital photography and high-definition video recordings. A system of checks was established where at minimum 10% of the data was repeated to ensure accuracy and determine measurement repeatability and single sample uncertainties.

Numerical Modeling

A three-dimensional numerical model (example shown in Fig. 1) was created using the commercially available CFD solver, Flow-3D (by Flow Science). Flow-3D solves the Reynolds-average Navier-Stokes equations (RANS) with modified algorithms to track the free surface, model flow past solid objects (in this case a labyrinth spillway), model the volume fraction of fluid in each discretized cell (VOF model), and includes multiple turbulence model options. Numerical modelling conducted in this study utilized the Renormalized Group (RNG) model with the exception of one numerical model that used the Large Eddie Simulation (LES) for a cursory comparison of turbulence model performance. The labyrinth weir was drafted as a three-dimensional solid object in AutoCAD and then imported into Flow-3D. The Fractional Area/Volume Obstacle Representation (FAVOR) algorithm was used to embed the labyrinth weir in the computational mesh.

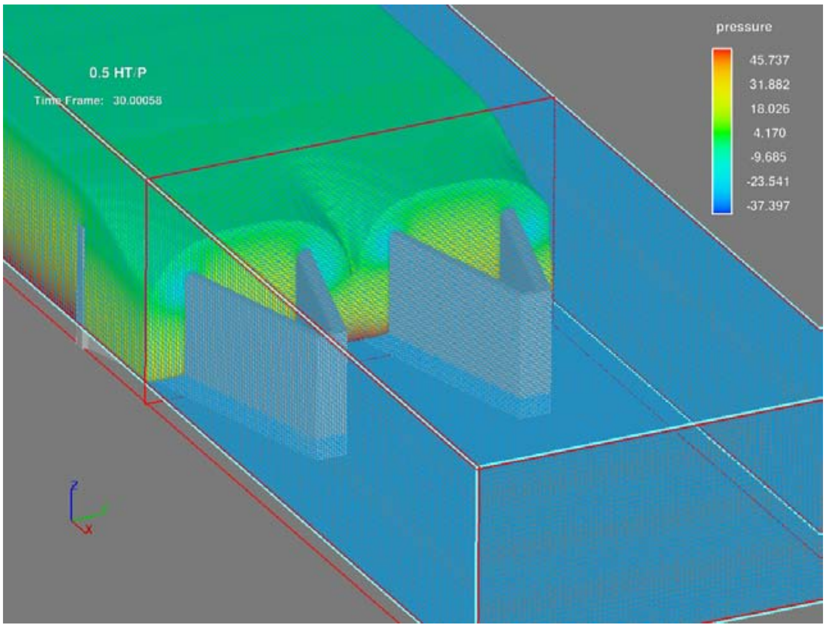

Fig. 2 – Numerical model, including computational mesh, boundary surfaces, and simulated fluid

The computational flow domain was defined based upon the physical models and discretized into hexahedral cells. Appropriate boundary conditions for the flow domain were defined (specified pressures, walls, etc). The simulation times were set to exceed 20 characteristic time scales (30 seconds). Selection of a computational mesh is critical to resolve the geometry and flow features and minimize simulation run time. A single uniform mesh was chosen (no embedded mesh blocks) to minimize numerical interpolation and truncation at mesh block boundaries. Multiple iterations were conducted to refine the mesh (e.g., P = 25 cells) and initial conditions, and to ensure sufficient run time and acceptable runtime diagnostics (e.g., stability limit, time step size) to reach a steady-state solution (tracked with flow convergence time history plots). Finalized simulation flow domains were comprised of 2.5 to 3+ million cells.

EXPERIMENTAL RESULTS

Head-Discharge Results

The physical model experimental data sets include 50 and 70 points (models 1 and 2, respectively). Six points were selected for comparison with numerical modeling. The numerical models were set with pressure boundaries and the CFD solver determined corresponding flow rates. As previously mentioned, simulations were repeated to refine the numerical models and ensure steady state conditions were reached. A total of 25 simulations were conducted. The $C_{d(\alpha°)}$ values determined from the physical modelling and the preliminary numerical model results are presented in Fig. (3). The α = 15° curve fit equation presented in Crookston (2010) is also included for comparison. It should be noted that this equation is based upon the model 2 experimental data set.

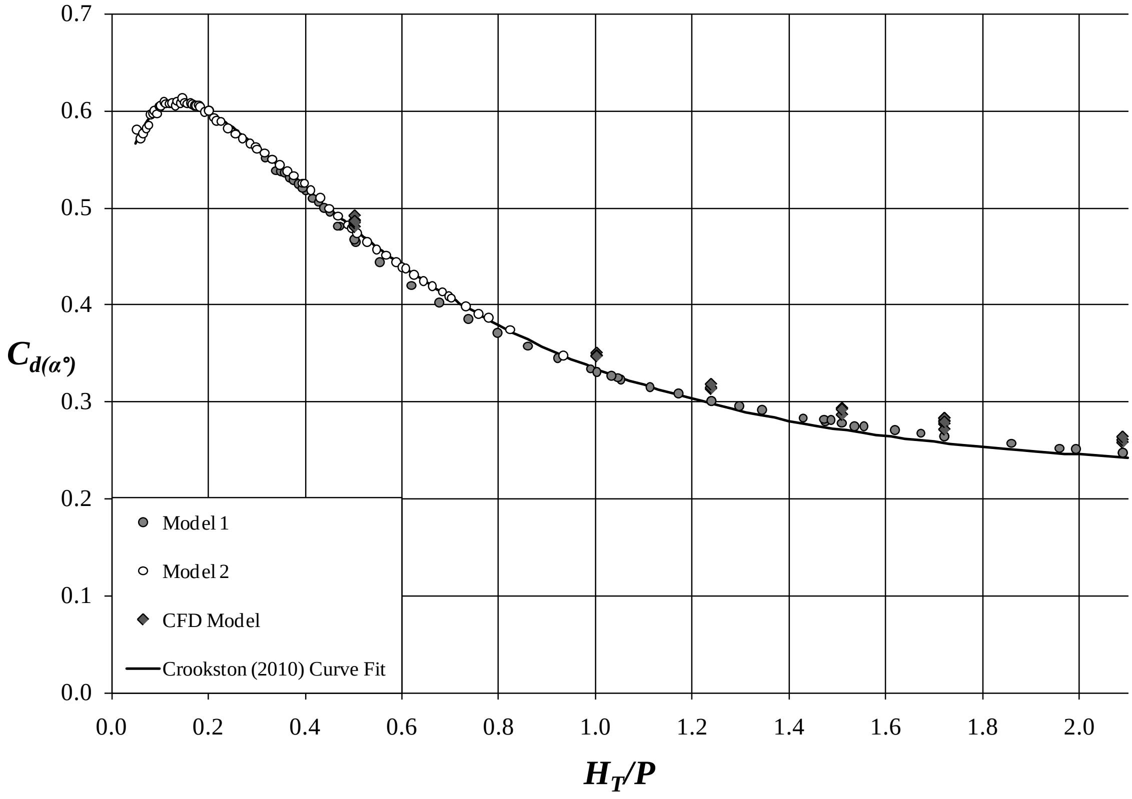

Fig. 3 - $C_{d(\alpha°)}$ vs. H_T/P for 15° quarter-round trapezoidal labyrinth weir, including preliminary numerical model results

As shown in Fig. (3), good agreement exists between the experimental results of models 1 and 2. The experimental results of model 1 also validate the α = 15° curve fit equation presented in Crookston (2010). The agreement between calculated $C_{d(\alpha°)}$ values from the numerical simulations, model 1, and the curve fit equation appear acceptable but merit a closer examination, presented in Fig. (4).

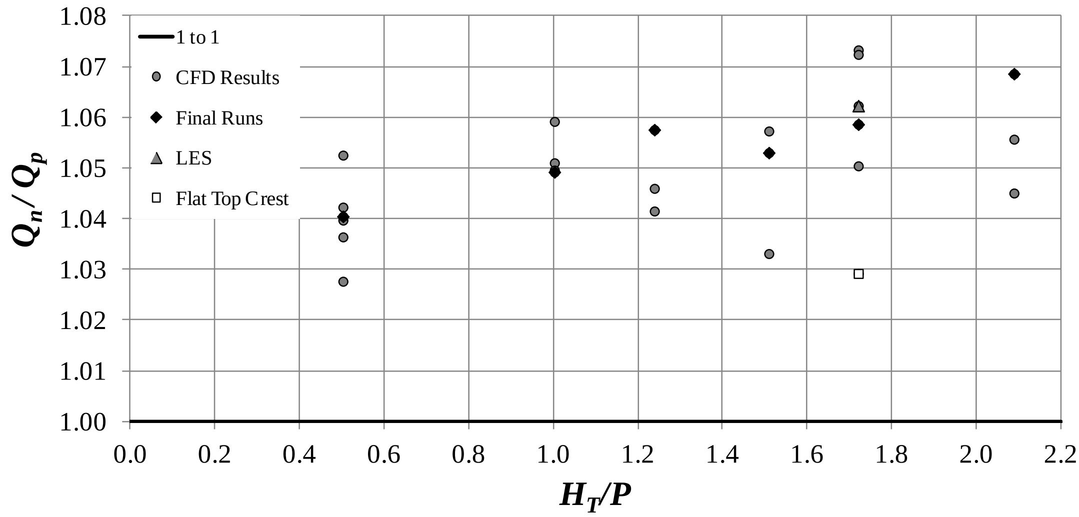

Fig. 4 – Agreement between Q_n and Q_p for 15° quarter-round trapezoidal labyrinth weir

Fig. (4) presents the ratio of numerically simulated flow rates (Q_n) to the flow rates measured in the physical model 1 (Q_p). The variation in simulations is displayed, with the finalized simulation depicted as solid black points. The final simulations are within 3% to 7% agreement, slightly over predicting labyrinth weir discharge. All runs except for one utilized the RNG turbulence model; although no conclusion can yet be made, the numerical differences due to the selection of the RNG or LES turbulence models at H_T/P = 1.72 is negligible. The 3-dimensional labyrinth weir was drafted in AutoCAD with a quarter-round crest shape to duplicate the physical model. However, a closer examination of the FAVOR algorithm shows that the model embedded in the hexahedral computational mesh is depicted more as a half-round type crest. Therefore, a single run was conducted with the sole difference being the imported geometry had a flat crest, which reduced the crest curvatures created by the FAVOR algorithm. This technique seems to better model the quarter-round crest shape and produce improved results (< 3%).

SUMMARY AND CONCLUSIONS

The physical modelling results validate the α = 15° curve fit equation presented by Crookston (2010) for H_T/P values exceeding 2.0. The preliminary numerical results indicate that for even high headwater ratios, CFD is an acceptable tool to examine the discharge performance of labyrinth weirs. Agreement between the physical modelling and numerical modelling is 3% to 7%. The results underscore the art associated with numerical modelling and 'rules of thumb' acquired from simulation experience. In addition to deviating from the physical geometry to better represent the quarter round crest shape with the FAVOR algorithm, the authors plan to conduct additional simulations to examine the effects of the upstream and downstream boundary conditions, specifically the proximity of the boundaries to the labyrinth weir. The authors believe, based upon the simulated water surface profiles, that improved agreement will result by moving the upstream boundary further from the spillway. It is believed that final simulation results within 3% to 4% of the physical model are obtainable.

In addition, the authors believe there is merit to numerical simulations of larger scaled labyrinth models, such as simulating model 2 (size scale = 2) and even prototype scale (size scale ≈ 36). The authors have collected, although limited to ≤0.02 $H_T/P \leq 0.37$, experimental data for a geometrically similar labyrinth weir that is 6 times larger than model 1 (P = 1 m). This data, obtained to examine size scale effect, shows excellent agreement with models 1 and 2, the curve fit equation, and would serve as an acceptable baseline for CFD validation. Finally, there is merit to validating at higher heads the remaining 6 quarter round curve fit equations (α = 6°, 8°, 10°, 12°, 20°, 35°) presented by Crookston (2010) and exploring numerical agreement between additional published experimental data sets.

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